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# Towards Effective Clustering Techniques for the Analysis of Electric Power Grids

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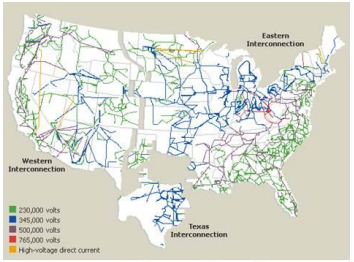
# Outline



- Overview
- Clustering Approaches
  - Fiedler vector
  - Evolutionary algorithm
- Case Studies
  - IEEE 50 generator system
  - Polish system
- Applications of Clustering

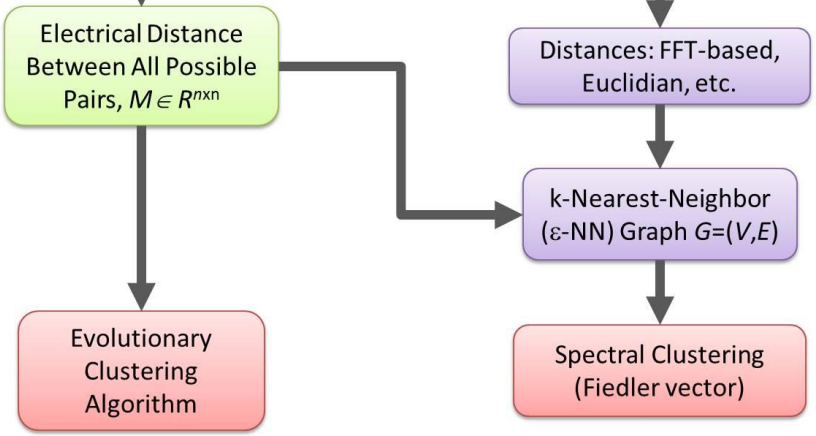
# Overview

- Two types of clustering
  - Evolutionary
  - Spectral
- Two types of data
  - Topological – uses information about connections in the grid
  - Derived – reporting from within the grid
- Two data sources
  - Electrical distance → topological
  - PMU data → derived




Topological and Electrical Structure

PMU Time series data



# Overview (cont.)



- **Goal**: group generators which have similar behavior after a disturbance
- Clustering from topological data groups generators together based on their proximity and connectivity
  - Predicts which generators should have similar behavior
- Clustering from derived data uses the observed behavior of the generators to cluster
  - Should be more reliable, but relies on data from after the disturbance
- We compare the results of one type of clustering on topological data with another type of clustering on derived data

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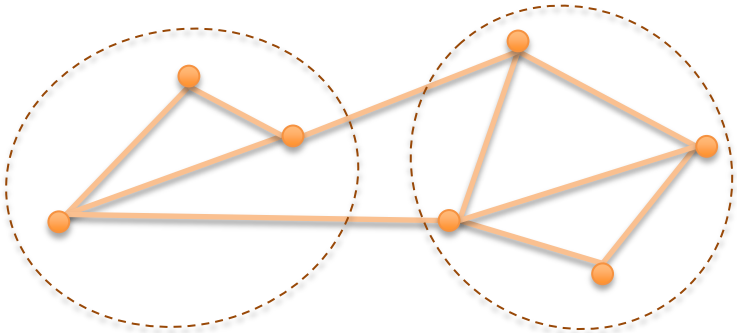
# Fiedler Clustering

## Introduction

- Input:
  - Graph – $G = (V, E, w)$ where $V$ is a set of entities, $E$ is a set of relationships between elements of $V$, $w$ is a weight function on the edges indicating a level of similarity between connected entities
  - Integer $k$ indicating number of clusters to break $V$ into
- Output:
  - Partition of $V$ into $k$ groups, $\langle V_1, V_2, \ldots, V_k \rangle$, so that their union is $V$ and $V_i \cap V_j = \emptyset$ for all $i \neq j$.
    - Two elements within the same partition should be more similar than two elements in different partitions



# Fiedler Clustering

## Background

- Given a weighted graph we calculate the weighted Laplacian,
  - Let $d_i = \sum_{(i,j)\in E} w(i,j)$, then the Laplacian is defined as

$$L(G) = \left(L_{i,j}\right)_{i,j=1..n} \text{ where } L_{i,j} = \begin{cases} d_i & i = j \\ -w(i,j) & (i,j) \in E \\ 0 & (i,j) \notin E \end{cases}$$

- Smallest eigenvalue is always zero (rows sum to zero)
  - Number of zero eigenvalues indicates number of connected components in the graph
- Eigenvector associated to smallest non-zero eigenvalue is called the *Fiedler vector*
  - Vertices make a continuous "choice" between -1 and +1, $-1 \leq f_i \leq 1$
  - Partition vertices into two groups – negative and positive
  - Zero values split between the two groups arbitrarily

# Fiedler Clustering

## Algorithm

- Input: $G = (V, E, w)$, $k \in \mathbb{N}$
- $P \leftarrow \{V\}$ Partition of vertices
- for each $p$ in $P$ until $|P| = k$
  - $L \leftarrow L(G[p])$ Laplacian of graph restricted to vertices in p
  - $F \leftarrow fielder(L)$
  - $p_- \leftarrow$ vertices with negative Fiedlers in $F$ (and some zeros)
  - $p_+ \leftarrow$ vertices with positive Fiedlers in $F$ (and some zeros)
  - $P \leftarrow P \setminus \{p\} \cup \{p_-, p_+\}$ Replace p with the partition of p into two sets
- RETURN $P$

- Some options:
  - How to split the vertices based on the Fiedler vector? Positive and negative as we have here, or less than midpoint vs. greater than midpoint?
  - How to pick the next element of $P$ to split?

# Evolutionary Clustering

## Electrically Cohesive Clusters



- Electrical distance,
  - Relationship between electric power transaction and voltage phase angle difference.
  - For a network of resistors, conductance matrix $G$,

$$I_a = \sum_{b=1}^{n} g_{ab} V_b \qquad \mathbf{V}_{\bar{r}} = \mathbf{G}_{\bar{r}\bar{r}}^{-1} \mathbf{I}_{\bar{r}}.$$

$$e(a,b) = g_{a,a}^{-1} + g_{b,b}^{-1} - g_{a,b}^{-1} - g_{b,a}^{-1}$$

  - For a power system, quadrant of the PF Jacobian,

$$\Delta \mathbf{P} = \left[ \frac{\partial P}{\partial \theta} \right] \Delta \theta$$

# Evolutionary Clustering

## Measures of clustering quality

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Electrical Cohesiveness Index, ECI (i.e., tightness index)

$$ECI = 1 - \frac{\hat{e}(C)}{\hat{e}_{max}} = 1 - \frac{\sum_{i=1}^{n} \sum_{j \in M_i} e_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij}}$$

Between-cluster Connectedness Index, BCCI

$$BCCI = 1 - \frac{h(C)}{h_{max}} = 1 - \frac{\sum_{i=1}^{n} \sum_{j \notin M_i} 1/e_{ij}}{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} 1/e_{ij}}$$

Cluster Count Index, CCI

$$CCI = e^{\frac{-(\ln p - \ln p_\star)^2}{2\sigma^2}}$$

Cluster Size Index, CSI

$$CSI = e^{\frac{-(\ln \bar{s} - \ln s_\star)^2}{2\sigma^2}}$$

Connectedness Index, CC

$$CC = \begin{cases} 1 & clusters\ are\ connected \\ 0 & clusters\ are\ not\ connected \end{cases}$$

Aggregate clustering fitness, f

$$f = ECI^{\alpha} \cdot BCCI^{\beta} \cdot CCI^{\gamma} \cdot CSI^{\zeta} \cdot CC$$

# Evolutionary Clustering

## GA configuration and parameters

| Parameter | Value |
|---|---|
| Representation | String of integers indicating connections |
| Fitness function | $f = ECI \times BCCI \times CSI \times CCI$ |
| Recombination method | 1-point crossover |
| Recombination probability | 80% |
| Mutation method | Random resetting with the feasible set $\{0,1,\ldots,m_i\}\ \forall i$ |
| Mutation probability, $p_m$ | $1/n$ |
| Parent selection | Tournament selection without replacement |
| Survival selection | Roulette wheel with elitism |
| Population size | 400 |
| Initialization | Randomly generate clusters |
| Termination condition | Run until sufficient quality is achieved |

# Evolutionary Clustering

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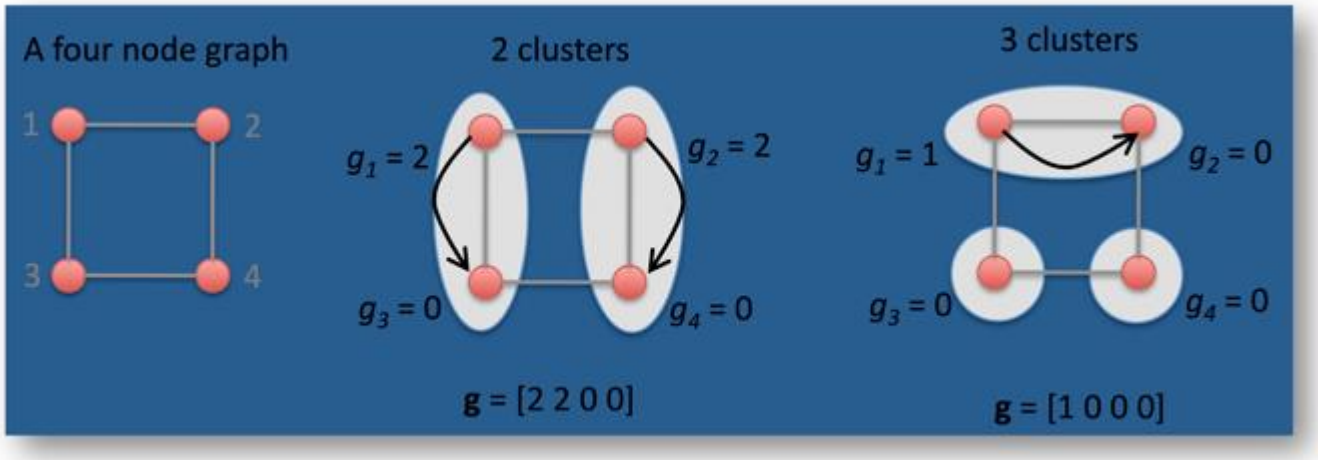

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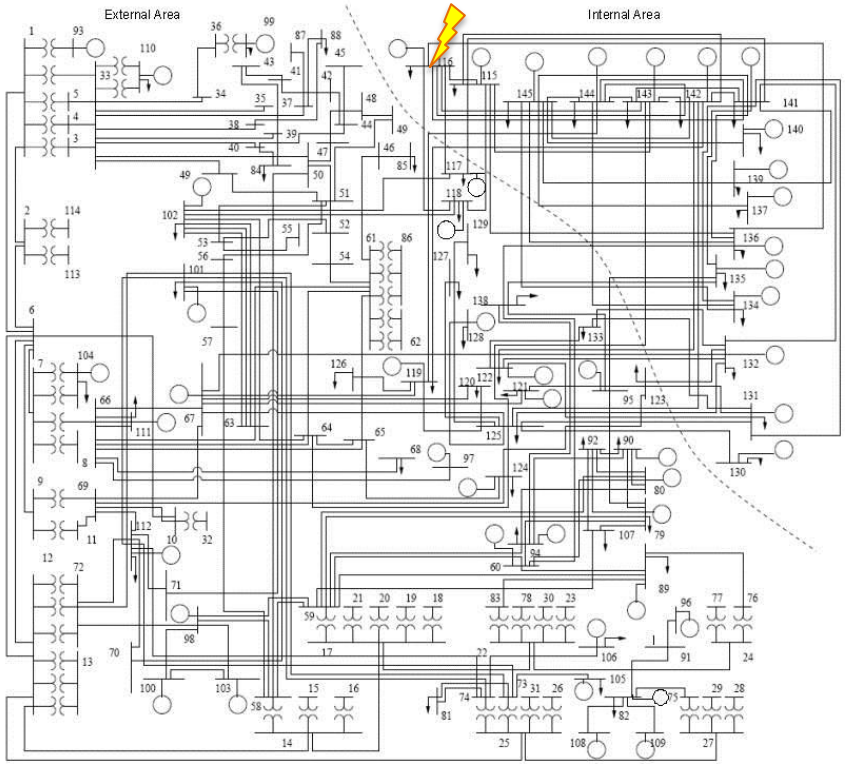
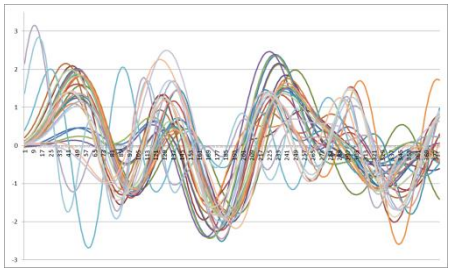
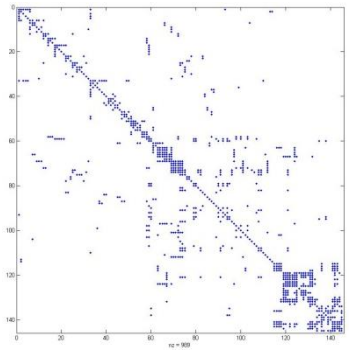
# IEEE 50 generator system

- 145-buses, 50-generators
- Internal area has 16 generators, external has 34
- Generators modeled using classical machine dynamics with 2nd order swing equation.
- Short-circuit fault at Bus 116 on line 116–136 for 60ms, then line tripped
- Data:
  - Oscillations of rotor angles in external system generators
  - $Y_{bus}$ matrix → electrical distance



Rotor angle time series for each generator



Nonzeros in $Y_{bus}$ matrix

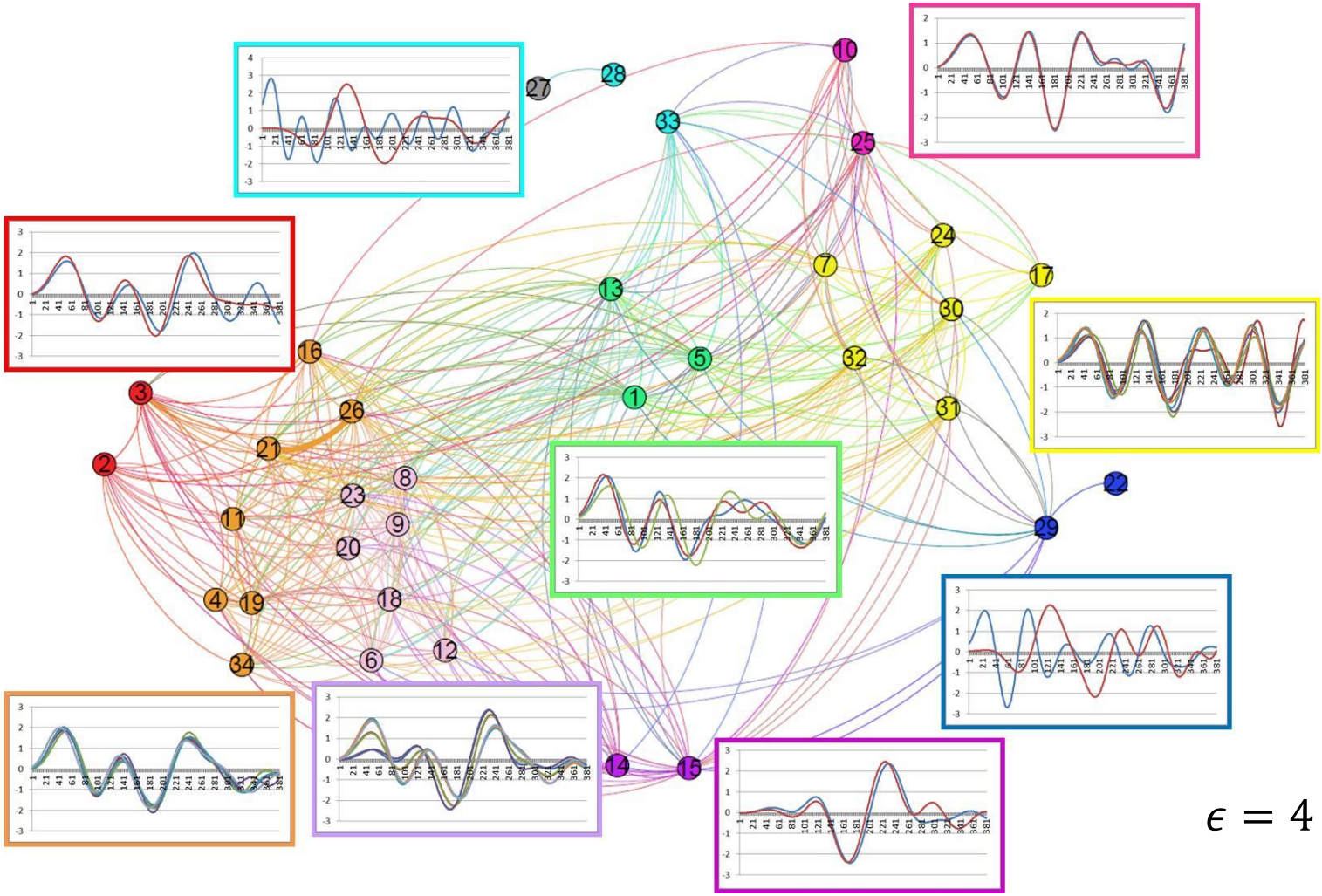
# Fiedler cluster, IEEE 50

## Setup

- Algorithm requires a graph: vertices are generators, what are edges?
  - Create $\epsilon$-nearest-neighbor graph from rotor angle data
  - Need notion of distance between generators based on their rotor angle time series
    - pseudo-distance based on discrete Fourier transform of time series vector
    - Euclidean distance between phase vectors of the DFT, weighted by absolute value of difference between amplitude vectors
  - $(v_i, v_j) \in E$ if the distance is less than some specified $\epsilon$
    - Larger or smaller $\epsilon$ will generate more or less sparse graph
  - Weights must be similarities, not distances, so
    - $w(v_i, v_j) = \frac{1}{d(v_i, v_j)}$ if there is an edge between $v_i$ and $v_j$
- Set number of clusters, $k = 10$

# Fiedler cluster, IEEE 50

## Results



$\epsilon = 4$

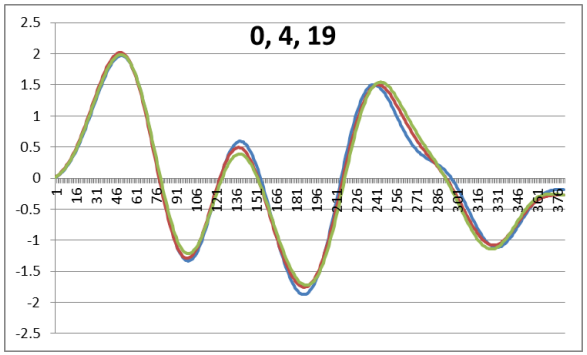
# Evolutionary clustering, IEEE 50

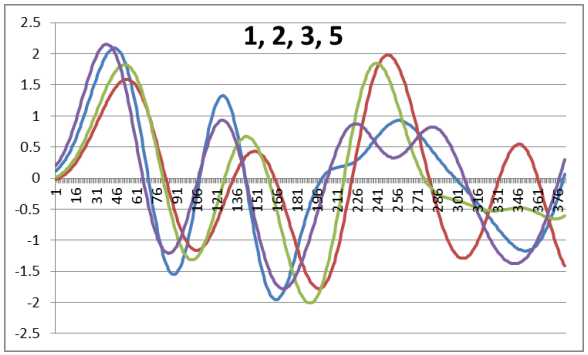
## Setup

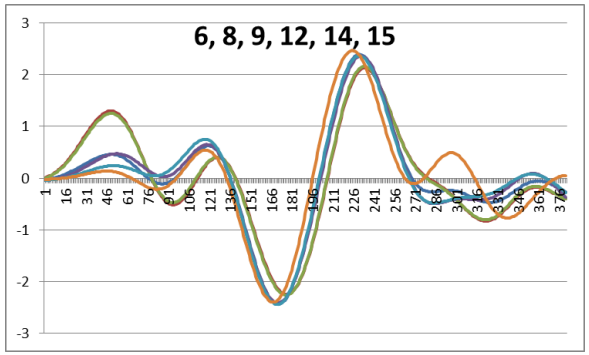
- Algorithm is designed for clustering buses rather than generators
- System has 145 buses – generators connect to 50 of those buses
- We cluster the generators based on which cluster its bus is in
- Use the electrical distance matrix, derived from the $Y_{bus}$

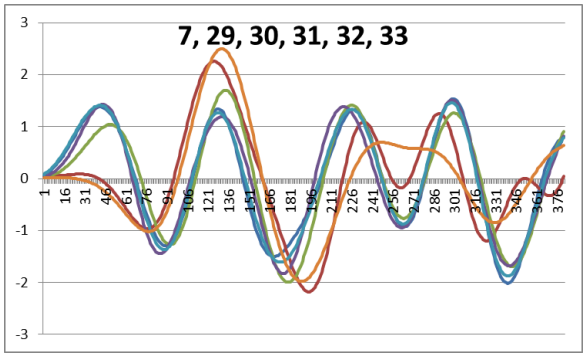
# Evolutionary clustering, IEEE 50

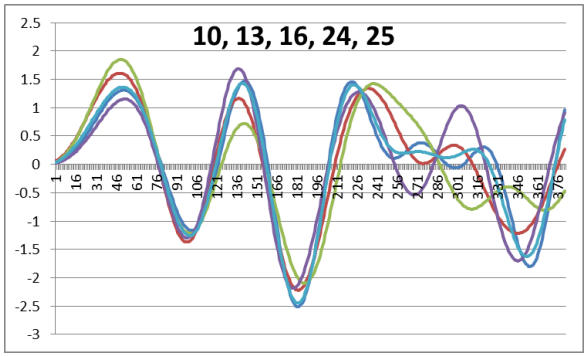
## Results

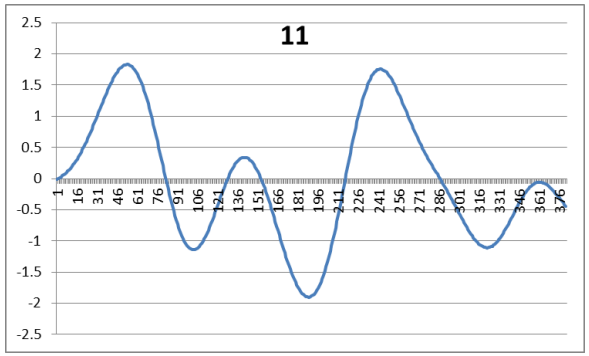


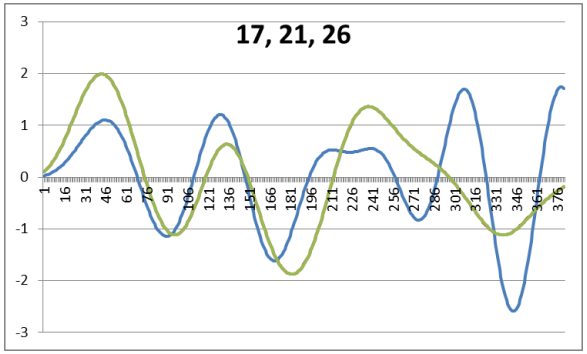



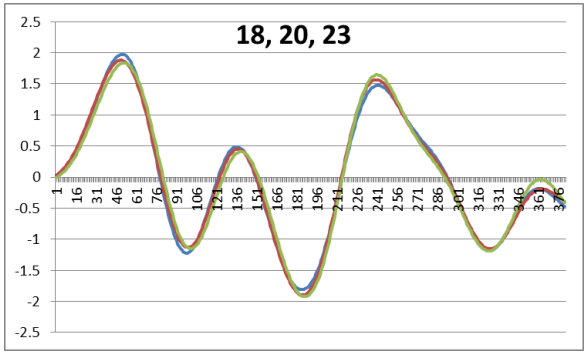



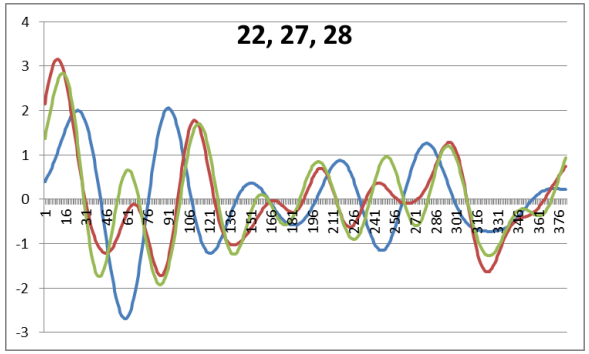



# Comparison



- Visual inspection of the time series for each clustering method shows that both do reasonably well
  - Both have some clusters that don't appear to be good fits
  - Both have many clusters that appear to behave very similarly
- Clusters from Fiedler method generally intersect a small number of clusters from evolutionary method
  - Generally taking a large portion of one cluster and smaller pieces of other clusters.

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# Applications



- Model reduction
  - Grouping generators that swing together and selecting a characteristic generator from each group to represent that group's behavior
- Defensive Islanding
  - Controlled partitioning as response to instabilities
  - Isolate self-sufficient portions (islands)
  - Incorporate electrical characteristics to identify portions that could be self-sufficient
- Market Power Potential
  - Small group of generators increase their revenue while others remain constant
  - Clustering can identify generators with market power
- Optimal placement of PMUs
  - PMU installation is expensive
  - Use clustering methods to identify groups that can share a PMU

# Future Work



- Exploring other types of spectral clustering
- Further comparison with other clustering methods
- Implementation of some of the applications mentioned

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